THE USE OF LEVEL SINGULAR VALUE DECOMPOSITION TECHNIQUES FOR VECTOR VELOCITY DETERMINATIONS AND ITS APPLICATION TO TID OBSERVATIONS

L.-C. Tsai*,**, J. Y. Liu*,**, F. T. Berkey*** and G. S. Stiles†

*Center for Space and Remote Sensing Research, National Central University, Chung-Li, Taiwan 32054
**Graduate Institute of Space Science, National Central University, Chung-Li, Taiwan 32054
***Space Dynamics Laboratory, Utah State University, Logan, Utah 84322-4145, U.S.A.
†Department of Electrical and Computer Engineering, Utah State University, Logan, Utah 84322-4120, U.S.A.

ABSTRACT

Building on previous work related to a determination of dynasonde phase parameters, we have obtained <2° standard deviation in the measured echo phases from an instrument at Bear Lake Observatory in Utah. These small uncertainties in phase measurement enable high-resolution echolocation components and line-of-sight Doppler velocity to be obtained. The purpose of this paper is to describe a level singular value decomposition technique to determine either the full vector velocity versus height or the plasma frequency in the ionosphere. Each three-dimensional velocity vector may be derived by a best fit to a set of echolocation components and Doppler velocities of spatially distributed echoes in independent steps over a frequency range of 0.2 MHz. Previous work has shown that ionospheric echoes in spread F can be used to derive a vector velocity that can be attributed to the flow of F-region plasma; in this work, data from the mid-latitude ionosphere are analyzed, which suggest that the three-dimensional phase motion of atmospheric gravity waves is responsible for the wave-like features that are observed.

DETERMINATION OF THE DYNASONDE PHASE PARAMETERS

The NOAA HF radar (dynasonde) utilizes an interferometric array and four-pulse, two-receiver set pattern to receive ionospherically reflected echoes. Phase parameters corresponding to phase changes due to antenna spacing, antenna orientation, sounding time interval, and sounding frequency increment are used to derive echo location, wave polarization, Doppler shift, and virtual range, respectively. Since 2π aliasing is inherent in interferometric phase measurements, the phase parameters cannot be derived directly from eight measured phases by the method of least squares. We have improved the derivation of these phase parameters by using a three step method [Tsai et al., 1993a]: (1) employing a "zero-freedom" technique to derive initial estimates of the phase parameters; (2) deriving shifted values of the measured phases from the first estimates, and; (3) using the method of least squares in conjunction with the shifted phases to improve the phase parameter estimates. This procedure minimizes the phase ambiguity inherent in interferometric phase measurements and derives phase parameters that approach the ideal least squares result. Furthermore, two error parameters, defined as ơ, the least squares of the measured phase errors, and EP, the rms phase error, are used to qualify a phase measurement. The theoretical value of ơ relative to the square of the standard deviation (σ) of each measured phase is equal.
to the number of degrees of freedom, and the distribution of $EP$ should be Gaussian for one degree of freedom, a Rayleigh distribution for two degrees of freedom, and so on. Figures 1a and 1b illustrate a first hop ionogram (including ordinary and extraordinary waves) recorded on 7 March 1993 and its corresponding values of $EP$. Analysis results determine an approximate standard deviation of $EP$ between $0.4^\circ$ and $1.0^\circ$, depending on the signal propagation (one hop or two hops) and the sounding time (day or night). For a standard deviation of $1.0^\circ$ for $EP$, we obtained a $\sim 2^\circ$ uncertainty in phase measurement for the Bear Lake Observatory instrument. The corresponding mean echolocation components and line-of-sight Doppler velocities within a frequency range of 0.2 MHz are shown in Figures 1c and 1d, respectively.

DETERMINATION OF VECTOR VELOCITIES

The dynasonde observation of ionospheric dynamics was first performed by Wright and Pitteway [1982]. They defined a traditional ionogram but with Doppler information added, denoting them 'dopplionograms' and interpreting the data to provide information on dynamic processes such as the effects of winds and waves in the plasma. Wright and Pitteway [1994] also developed a procedure of vector velocity estimation based on least squares weighted according to an rms phase error derived by the various echo phases received in a dipole array during one pulse set. In attempting to solve the same least squares problem, Jarvis [1995] applied singular value decomposition [Forsythe et al., 1977] to measure vector velocities from observations on the Advanced Ionospheric Sounder (AIS) at Halley, Antarctica. The AIS is also a NOAA III radar or dynasonde utilizing a different pulse set configuration and antenna array to the dynasonde used by Wright and Pitteway [Tsai et al., 1993b]. However, both studies used an entire recording or parts of it suitably selected according to phase error. For each echo, a vector velocity $V = (V_x, V_y, V_z)$ is determined by

$$V^* = \frac{L_x}{R} V_x + \frac{L_y}{R} V_y + \frac{L_z}{R} V_z,$$

where $V^*$ is a line-of-sight Doppler velocity, $(L_x, L_y, L_z)$ are three-dimensional echolocation positions, and $R$ is line-of-sight echo range. An ensemble of $N$ echoes may share the same $V$ of the radio reflection surface. Theoretically, a statistical best fit velocity vector can be derived from echolocation components and Doppler velocities of many spatially distributed ionospheric echoes, where the number of data points
is much larger than three. In solving least squares problems where Gaussian elimination and LU decomposition fail to give satisfactory results when sets of linear equations are either singular or else numerically very close to singular, singular value decomposition (SVD) will be proposed as an appropriate technique in this paper.

With reference to previous work on the determination of vector velocities using 'spread' echoes from the E- and F-region with Fresnel-sized irregularities, this paper concentrates on the analysis of 'clean' echoes (no spread) to observe traveling ionospheric disturbances (TIDs), with applications to atmospheric gravity wave parameter estimation. It is now generally accepted that the traveling ionospheric disturbances are excited by propagating acoustic gravity waves in the neutral atmosphere. Theoretical treatments of gravity wave propagation [e.g. Row, 1967; Francis, 1974] suggest that the disturbance does not have a unique frequency or wavelength and occurs over a finite height. The vector velocity results are, therefore, calculated on linked records in independent steps over a frequency range of 0.2 MHz, and the analysis is termed 'level' singular value decomposition. The corresponding relationship between plasma frequency/electron density and height can be derived by true-height analysis [Tsai et al., 1995]. In solving the linear system of equations $Ax = b$ by SVD, the condition number $\kappa$ of the nonsingular matrix $A$ with singular values $\sigma_1 \geq \ldots \geq \sigma_n \geq 0$ is defined to be the ratio $\sigma_1/\sigma_n$. It is well known that the nonzero singular values will correspond to the positive square roots of the eigenvalues of the nonnegative Hermitian matrices $A^H A$ and $A A^T$. The condition number will give a measure of how much errors in $A$ and/or $b$ may be magnified in the computed solution of $x$. Therefore, the corresponding error in the velocity components shown in Figure 2 is determined by the production of the condition number and an error of $\varepsilon$ in the estimation of Doppler velocity, where $\varepsilon$ is determined by the derivation of the $\text{EP}$ in the analysis.

APPLICATIONS TO TID OBSERVATION

The vector velocity determination assumes that the Doppler velocity $V^*$, which is derived from rate of change of phase path for ionospheric echoes, is solely due to motion of the reflecting iso-electron density surface and is not due to a change of electron density along the signal path. Using the reflection assumption, Jarvis [1995] derived vector velocities from spread echoes which were consistent with other measurements of the background plasma flow in the F-region ionosphere. On the other hand, using 'clean' ionospheric observations, it is difficult to ascertain whether the derived vector velocity is the mass motion of ionization clouds or the phase velocity of a gravity wave. There is considerable evidence that the ionosphere often acts as a smooth reflector with a wave-like structure and the diffraction pattern is produced by rays reflected from different parts of the structure [Vincent, 1972]. Other evidence of the existence of an internal gravity wave in our observations has been investigated by the comparison between iso-frequency virtual height profiles and Doppler velocity profiles with time. As shown in Figure 3 various temporal profiles were obtained on a 150 min segment from 2006 UT to 2233 UT on March 7, 1993, at Bear Lake, Utah, with 3 min time resolution. With increasing virtual heights the corresponding
Doppler velocities are positive, and vice versa. Applying the Maximum Entropy Method of spectral analysis to the iso-frequency virtual height profiles during this interval, evidence for an atmospheric gravity wave (AGW) with a period of about 75 min was obtained. Once the wave period/frequency and the vector velocity of an AGW are known, the corresponding wave parameters may be calculated.

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REFERENCES


